ADDING FRACTIONS

BE CAREFUL !!
THINK ABOUT IT !!

WE CAN ONLY ADD / SUBTRACT FRACTIONS IF THE NUMBERS ON THE BOTTOM ARE THE SAME.

eg $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

(DENOMINATORS)

IF THE NUMBERS ON THE BOTTOM ARE NOT THE SAME:

- WE NEED TO MAKE THEM THE SAME.
- TO DO THIS, RE-WRITE THE QUESTION AS A SINGLE FRACTION.

STEP 1: WORK OUT THE BOTTOM LINE

$$\frac{1}{5} + \frac{2}{3}$$

$$= \frac{??????}{15}$$

3	5
6	10
9	15
12	
15	

SNAP!

THIS IS THE LOWEST COMMON DENOMINATOR (L.C.D.)

STEP 2: WORK OUT THE TOP LINE

SO WHAT GOES ON TOP?

DIVIDE THE NEW BOTTOM NUMBER BY THE BOTTOM OF EACH OF THE ORIGINAL FRACTIONS, AND PUT THAT ANSWER OUTSIDE A BRACKET ON THE TOP LINE

$$\frac{1}{5} + \frac{2}{3}$$

$$= \frac{3(\quad) + 5(\quad)}{15}$$

5 INTO 15 GOES 3 TIMES

Q. WHAT GOES INSIDE THE BRACKETS ??

A. THE TOP LINE OF EACH OF THE ORIGINAL FRACTIONS.

STEP 3:

$$\frac{1}{5} + \frac{2}{3}$$

$$= \frac{3(1) + 5(2)}{15} \longrightarrow \frac{3 + 10}{15} = \boxed{\frac{13}{15}}$$

WHAT IF THERE ARE LETTERS AS WELL AS NUMBERS ??

IT'S THE SAME METHOD ! USE THE STEPS + INSTRUCTIONS FROM THE PREVIOUS PAGE TO FOLLOW THESE EXAMPLES.

eg ①

$$\frac{x}{2} + \frac{x}{5}$$

L.C.D = 10

$$= \frac{5(x) + 2(x)}{10}$$

$$= \frac{5x + 2x}{10} = \boxed{\frac{7x}{10}}$$

eg ②

$$\frac{3x - 2}{4} + \frac{2x + 1}{3}$$

L.C.D = 12

$$= \frac{3(3x - 2) + 4(2x + 1)}{12}$$

$$= \frac{9x - 6 + 8x + 4}{12}$$

$$= \boxed{\frac{17x - 2}{12}}$$

ALGEBRAIC FRACTIONS (UGH - EVEN WORSE THAN FRACTIONS!)

Q. WHAT HAPPENS WHEN THE LETTERS ARE ON THE BOTTOM? A. EXACTLY THE SAME.

IF IT'S JUST ONE FRACTION:

- FACTORISE IF YOU CAN
- DIVIDE TOP AND THE BOTTOM BY SAME THING
- OR DIVIDE TOP BY THE BOTTOM.

eg ① $\frac{a^7}{a^3} = a^4$

② $\frac{20a^5}{15a^3} = \frac{4a^5}{3a^3}$ ← DIVIDE TOP + BOT BY 5

$= \frac{4a^2}{3}$ ← DIVIDE a^5 by a^3

FACTORISING → SEE NOTES ON FACTORISING

③ $\frac{x^2 + 6x + 8}{x+4} \longrightarrow \frac{\cancel{(x+4)}(x+2)}{\cancel{(x+4)}}$

$= \boxed{x+2}$

IF THERE'S MORE THAN ONE FRACTION,

FIND L.C.D. [NOW, WE CAN'T REALLY FIND L.C.D. OF LETTERS, SO WE MULTIPLY THEM TOGETHER]

eg WRITE

$\frac{1}{x+2} + \frac{1}{x}$ AS A SINGLE FRACTION.

L.C.D. $= (x+2)(x)$

$$\frac{x(1) + (x+2)(1)}{(x+2)(x)} = \frac{x + x + 2}{(x+2)(x)} = \frac{2x+2}{(x+2)(x)}$$

ALGEBRA (LINEAR)

- SOLVING LINEAR EQUATIONS

① TIDY UP
- REMOVE ANY BRACKETS
- GET RID OF FRACTIONS
- ADD LIKE TERMS

② GET x'S ON ONE SIDE [USUALLY LEFT]

③ GET NUMBERS ONTO OTHER SIDE.

④ DIVIDE BY NUMBER IN FRONT OF x TO SOLVE.

$$2(2x-4) = 12 - 3(2x-1)$$

①
$$4x - 8 = 12 - 6x + 3$$
$$4x - 8 = 15 - 6x$$
$$+6x \qquad\qquad +6x$$

I DON'T WANT x'S OVER HERE, SO ADD $6x$ TO BOTH SIDES

$$10x - 8 = 15$$
$$+8 \qquad\qquad +8$$

NEED TO GET RID OF THIS -8, SO ADD 8 TO BOTH SIDES

$$10x = 23$$
$$\div 10 \qquad\qquad \div 10$$

THIS MEANS "x MULTIPLIED BY 10" SO TO UNDO THIS I DIVIDE (BOTH SIDES) BY 10

$$\boxed{x = 2.3}$$

METHOD

① TIDY UP.
GET RID OF BRACKETS
ADD LIKE TERMS.

② GET x'S ON ONE SIDE

③ GET NUMBERS ONTO OTHER SIDE.

④ DIVIDE BY NUMBER IN FRONT OF x

WITH PRACTICE YOU WILL GET MUCH QUICKER AND WILL BE ABLE TO TAKE SHORT CUTS, BUT ONLY IF YOU UNDERSTAND WHY

GOLDEN RULES:

- I CAN ONLY WORK ON ONE SIDE OF THE EQUATION AT ANY ONE TIME.
- WHATEVER I DO TO ONE SIDE, I HAVE TO DO THE EXACT SAME THING TO THE OTHER SIDE...

EQUATIONS WITH FRACTIONS

→ UGH. GET RID...

TO GET RID OF FRACTIONS FROM OUR EQUATION WE MULTIPLY **EVERYTHING** BY THE LOWEST COMMON DENOMINATOR ← SNAP!!! WITH NUMBERS ON BOTTOM

eg
$$\frac{x}{6} - \frac{x}{2} = 5$$

① L.C.D. = 6
[MULTIPLY EACH TERM BY 6]

$$6\left(\frac{x}{6}\right) - 6\left(\frac{x}{2}\right) = 6(5)$$

② NOW DIVIDE EACH NUMBER ON THE BOTTOM INTO THE NUMBER OUTSIDE THE BRACKET.

$$\cancel{6}^{1}\left(\frac{x}{\cancel{6}}\right) - \cancel{6}^{3}\left(\frac{x}{\cancel{2}}\right) = 6(5)$$

③ TIDY UP

$$x - 3x = 30$$

$$-2x = 30$$ ← CHANGE SIGNS OF BOTH SIDES.

$$2x = -30$$

$$\boxed{x = -15}$$ ← DIVIDE BY 2

SIMULTANEOUS EQUATIONS

eg
$$2x + 5y = -15$$
$$4x + 3y = -9$$

PROBLEM: • WE HAVE 2 LETTERS: x's AND y's.

• WE CAN ONLY SOLVE EQUATIONS WITH ONE LETTER.

SOLUTION: WE NEED TO GET RID OF ONE OF THE LETTERS.

GREAT.... $\boxed{\text{HOW?}}$

— IF WE HAVE THE SAME NUMBER OF y's IN BOTH EQUATIONS, WITH 1 $+$ AND 1 $-$, THEN ADDING THE EQUATIONS TOGETHER WILL LEAVE ME WITH NO y's.... GREAT!!

— HOW? • MULTIPLY TOP LINE BY NUMBER IN FRONT OF y ON THE BOTTOM AND VICE VERSA.

• YOU MIGHT NEED TO CHANGE THE SIGN OF ONE ENTIRE EQUATION.

So, $\times 3 \rightarrow ①$ $2x + 5y = -15$

$\times 5 \rightarrow ②$ $4x + 3y = -9$

$\times 3$ ① $6x + 15y = -45$

$\times 5$ ② $20x + 15y = -45$ ← CHANGE SIGNS OF TOP LINE

$-6x - \cancel{15y} = 45$

$20x + \cancel{15y} = -45$ ADD ② EQUATIONS TOGETHER...

$14x = 0$ y's DISAPPEAR...

$x = 0$

CONTINUED ON NEXT PAGE...

$x = 0$

BUT WE ALSO NEED TO FIND y.

① $2x + 5y = -15$ ← CHOOSE EITHER OF YOUR FIRST TWO EQUATIONS

$2(0) + 5y = -15$ ← SUBSTITUTE IN YOUR ANSWER FOR x ($x = 0$)

$5y = -15$

$\boxed{y = -3}$

ANS: $\boxed{x = 0, \quad y = -3}$

YOU SOMETIMES NEED TO RE-ARRANGE THE INITIAL EQUATIONS TO LOOK LIKE THIS...

FOR EXAMPLE,
- GET RID OF BRACKETS / FRACTIONS
- GET x'S AND y'S ONTO SAME SIDE
- GET NUMBERS ONTO OTHER SIDE.

IN THEORY, WHEN YOU GET AN ANSWER FOR x AND y, THIS IS A "POINT" ON THE X AND Y AXES. THIS IS THE PLACE WHERE THE 2 <u>LINES</u> CROSS

eg $2x + 5y = -15$ IS A LINE...

INEQUALITIES

$<$ $>$ $\geqslant$ $\leqslant$

[DON'T WORRY ABOUT THE SIGNS...]

TREAT JUST LIKE EQUATIONS
BUT
NEVER CHANGE SIGNS OR MULTIPLY/DIVIDE BY A MINUS NUMBER.

eg

$$3x - 2 \leqslant 4$$

$$+2 \qquad +2$$

GET x's ON ONE SIDE
GET NUMBERS ON OTHER SIDE

$$3x \leqslant 6$$

÷ BY 3

$$\boxed{x \leqslant 2}$$

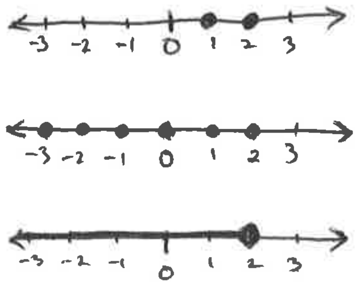
$x \leqslant 2$ LESS THAN OR EQUAL TO 2

GRAPHING INEQUALITIES

$x \in \mathbb{N}$

$x \in \mathbb{Z}$

$x \in \mathbb{R}$



$\mathbb{N}$ = NATURAL NUMBERS

$\mathbb{Z}$ = INTEGERS

[WHOLE NUMBERS - USE DOTS]

$\mathbb{R}$ = REAL

(USE HEAVY BLACK LINE)

MANIPULATING FORMULAS / REARRANGING EQUATIONS WITH LOTS OF LETTERS

- THESE LOOK HARD BUT ARE ACTUALLY EASIER THAN EQUATIONS BECAUSE THERE IS NO CALCULATION INVOLVED . . .

eg Express $\underline{t}$ in terms of u, v, and a.

$$v = u + at$$

SWITCHED SIDES TO GET t ON LEFT.

$$u + at = v$$

$$-u \qquad\qquad -u$$

NEED TO GET t ON ITS OWN

$$at = v - u$$

THIS IS NEARLY IT.

$$t = \frac{v-u}{a}$$

DIVIDE BY LETTER IN FRONT OF t

at means $t \times a$

UNDO $\times a$ SO $\div a$

METHOD

WE WANT TO GET $t = ?$

1. GET ANYTHING WITH t ON ITS OWN ON THE LEFT.
2. IN REALLY HARD QUESTIONS YOU WILL NEED TO TAKE OUT A FACTOR OF t.

WORD PROBLEMS (HARD)

- THEY GIVE YOU A "PROBLEM" IN WORDS. WE NEED TO SOMEHOW MAKE THIS INTO AN EQUATION AND SOLVE IT.

- READ QUESTION CAREFULLY

- HIGHLIGHT IMPORTANT WORDS.

- LET UNKNOWN NUMBER $= x$

- IF THERE ARE 2 UNKNOWNS, LET THE OTHER LETTER $= y$, THEN YOU WILL GET SIMULTANEOUS EQUATIONS...

- IF YOU'RE NOT SURE HOW TO FORM THE EQUATION, MAKE UP A NUMBER FOR x, AND WRITE DOWN HOW YOU WOULD WRITE DOWN THE EQUATION IF THIS WAS CORRECT.

eg Q. WHEN I MULTIPLY A NUMBER BY 12 AND ADD 37, THE RESULT IS 325. FIND THE NUMBER.

A.

NUMBER $= x$

PRETEND $x = 5$

MULTIPLY 5 BY 12 AND ADD 37

$5 \times 12 + 37$

BUT x IS NOT 5, SO WRITE THIS AS

$x \times 12 + 37 = 325$

SOLVE THIS

$12x + 37 = 325$

ALGEBRA (QUADRATICS)

- A QUADRATIC EXPRESSION HAS AN x^2 TERM

(IT COULD BE A DIFFERENT LETTER)

[QUAD BIKE = 4 WHEELS
SQUARE = 4 SIDES]

eg $x^2 - 3x - 4$ IS A QUADRATIC EXPRESSION

- A QUADRATIC EQUATION HAS AN "=" SIGN.

eg $x^2 - 3x - 4 = 0$

THE "= 0" BIT IS VERY VERY IMPORTANT

IF IT'S NOT =0 WE MIGHT NEED TO CHANGE THINGS AROUND TO GET A =0

WE HAVE 2 WAYS TO SOLVE QUADRATIC EQUATIONS

FACTORS	−b FORMULA
BE CAREFUL OF THE SIGNS	BE CAREFUL OF THE SIGNS

① MAKE SURE YOU GET AN "= 0" IN YOUR EQUATION

YOUR EQUATION SHOULD LOOK LIKE: $ax^2 + bx + c = 0$

② FACTORISE	OR	② USE THE FORMULA.
③ LET EACH BRACKET =0 AND SOLVE		③ THESE ARE YOUR 2 ANSWERS
④ THESE ARE YOUR 2 ANSWERS		

EITHER WAY, YOU SHOULD ALMOST ALWAYS GET 2 ANSWERS / SOLUTIONS. WE OFTEN CALL THESE THE **ROOTS** OF THE EQUATION.

FACTORISING

A EASIER ONES

BE CAREFUL

WATCH THOSE SIGNS !!

1. $x^2 + 3x + 2$	**2.** $x^2 + 4x + 3$	**3.** $x^2 + 6x + 5$
4. $x^2 + 8x + 7$	**5.** $x^2 + 12x + 11$	**6.** $x^2 + 6x + 8$
7. $x^2 + 5x + 4$	**8.** $x^2 + 7x + 12$	**9.** $x^2 + 7x + 10$
10. $x^2 + 11x + 10$	**11.** $x^2 + 8x + 12$	**12.** $x^2 + 13x + 12$
13. $x^2 - 9x + 14$	**14.** $x^2 - 10x + 21$	**15.** $x^2 - 8x + 12$
16. $x^2 - 2x - 8$	**17.** $x^2 + 8x - 20$	**18.** $x^2 - 4x - 12$
19. $x^2 + 2x - 15$	**20.** $x^2 - x - 12$	**21.** $x^2 + x - 30$

B HARDER ONES

1. $2x^2 + 5x + 3$	**2.** $3x^2 + 8x - 3$	**3.** $2x^2 - 7x + 6$
4. $3x^2 - 4x - 7$	**5.** $2x^2 - 9x - 5$	**6.** $5x^2 + 9x - 2$
7. $2x^2 + 7x - 15$	**8.** $3x^2 - 11x - 20$	**9.** $7x^2 + 5x - 2$

EXAMPLES

A 8. $x^2 + 7x + 12$

$x \quad 3 \qquad 3x$

$+$

$x \quad 4 \qquad 4x$

$7x$ ✓

$(x + 3)(x + 4)$

B 4. $3x^2 - 4x - 7$

$3x \quad -7 \qquad -7x$

$+$

$x \quad 1 \qquad 3x$

$-4x$ ✓

$(3x - 7)(x + 1)$

FACTORISING TO SOLVE A QUADRATIC EQUATION

eg $2x^2 + 15x + 7 = 0$

① MAKE SURE EQUATION HAS $=0$

$2x \quad\quad 1 \quad\quad 1x$

$x \quad\quad 7 \quad\quad 14x$

$15x$ ✓

② FACTORISE

$(2x+1)(x+7) = 0$

③ LET EACH FACTOR $=0$ + SOLVE

$2x + 1 = 0$
$\quad -1 \quad\; -1$

$2x = -1$

$\boxed{x = \frac{-1}{2}}$

$x + 7 = 0$

$\boxed{x = -7}$

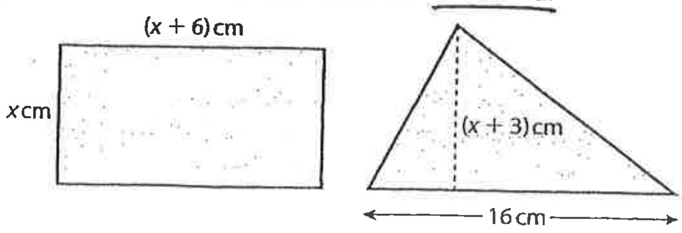
$\boxed{2 \quad \text{ROOTS}}$

THEY SOMETIMES LIKE TO HIDE THE QUADRATIC EQUATION, FOR EXAMPLE BY GIVING YOU A RECTANGLE / TRIANGLE AND ASKING YOU TO WORK OUT THE AREA. REMEMBER, ALGEBRA FOLLOWS THE SAME RULES AS NORMAL NUMBERS!

YOU WILL NEED TO RECOGNISE QUADRATIC EQUATIONS IN THESE UNFAMILIAR SITUATIONS.

SEE THE NEXT PAGE FOR AN EXAMPLE...

EG. The rectangle and triangle below each have the same area.



(i) Write an expression in x for

(a) the area of the rectangle (b) the area of the triangle.

(ii) Form an equation and solve it to find the value of x.
Hence find the dimensions of the rectangle.
Why did you take only one value for x?

(i)

(a) LENGTH × WIDTH

$x(x+6)$

$= x^2 + 6x$

(b) $\frac{1}{2}$ BASE × ⊥ HEIGHT

$= \frac{1}{2}(16)(x+3)$

$= 8(x+3)$

$= 8x + 24$

(ii) SAME AREA. SO

$$x^2 + 6x = \cancel{8x} + \cancel{24}$$
$$-8x \quad -24 \qquad -\cancel{8x} \quad -\cancel{24}$$

WE NEED AN $= 0$

$$x^2 - 2x - 24 = 0$$

$x \quad 4 \quad 4x$

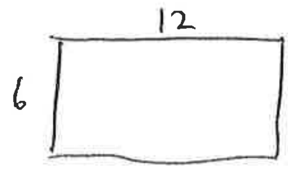
$x \quad -6 \quad -6x$

$-2x$ ✓

$$(x+4)(x-6) = 0$$

$\cancel{x = -4}$ $\quad x = 6$

RECTANGLE =



RULE OUT $x = -4$ BECAUSE A SIDE OF A RECTANGLE MUST BE A POSITIVE NUMBER.

THEY CAN ALSO HIDE QUADRATIC EQUATIONS IN EQUATIONS WITH FRACTIONS. FOLLOW ALL THE USUAL RULES FOR EQUATIONS WITH FRACTIONS.

$\boxed{\text{YOU WILL NEED TO RECOGNISE A QUADRATIC EQUATION}}$

eg
$$\frac{x+7}{3} + \frac{2}{x} = 4$$

L.C.M. = $3x$

SO MULTIPLY EACH TERM BY $3x$

$$\cancel{3}x\left(\frac{x+7}{\cancel{3}}\right) + 3\cancel{x}\left(\frac{2}{\cancel{x}}\right) = 3x\left(4\right)$$

$$\underset{-12x}{x^2 + 7x} + 6 = \underset{-12x}{12x}$$

WE NEED AN $= 0$

$$x^2 - 5x + 6 = 0$$

NOW SOLVE IN THE USUAL WAY.

SEE SECTION ON LINEAR EQUATIONS FOR OTHER EXAMPLES OF EQUATIONS WITH FRACTIONS

QUADRATIC FORMULA

- THE **OTHER METHOD** TO SOLVE QUADRATIC EQUATIONS.

- YOU NEED TO KNOW THIS, AS IT CAN SOMETIMES COME UP IN THE "COMPLEX NUMBERS" QUESTION. IT **CAN ALSO BE THE <u>ONLY</u> WAY** TO ANSWER CERTAIN QUADRATIC EQUATIONS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

SOME NOTES

1. THE FORMULA IS ON THE <u>FRONT COVER</u> OF THE TABLES BOOK!
2. BE CAREFUL WITH SIGNS.
3. YOU WILL GET 2 ANSWERS
4. $\pm$ MEANS "PLUS OR MINUS"

STEPS

① WRITE EQUATION IN CORRECT ORDER

eg $ax^2 + bx + c \underbrace{= 0}_{\uparrow \text{ IMPORTANT.}}$

② WRITE DOWN WHAT a= , b= , c= .
(THESE TAKE WHATEVER SIGN IS ATTACHED TO THEM IN THE EQUATION)

eg $3x^2 - 7x + 2 = 0$

$\boxed{a = 3 \quad b = -7 \quad c = 2}$

③ WORK OUT YOUR 2 ANSWERS. USE A CALCULATOR, BUT PLEASE... **WATCH THE SIGNS**

- IT CAN BE EASIER IF YOU DO THE $\sqrt{\quad}$ BIT FIRST, ON SOME ROUGH WORK

EXAMPLE ON NEXT PAGE...

eg $$3x^2 + 5x - 3 = 0$$

$a = 3 \quad b = 5 \quad c = -3$

$\pm$ MEANS YOU WILL GET 2 ANSWERS

1 $\oplus$ / 1 $\ominus$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-5 \pm \sqrt{(5)^2 - 4(3)(-3)}}{2(3)}$$

RW

$$\sqrt{(5)^2 - 4(3)(-3)}$$

$$= \sqrt{61}$$

$$\frac{-5 \pm \sqrt{61}}{6}$$

$$\frac{-5 + \sqrt{61}}{6} \quad \text{OR} \quad \frac{-5 - \sqrt{61}}{6}$$

$$\boxed{x = 0.468} \quad \text{OR} \quad \boxed{x = -2.135}$$

QUADRATIC EQUATIONS (WORKING BACKWARDS)

- SOMETIMES YOU WILL BE GIVEN THE ANSWERS / ROOTS AND BE ASKED TO FORM THE EQUATION.

FIRSTLY, LET'S LOOK AT WHAT THE "NORMAL" QUADRATIC EQUATION LOOKS LIKE

EQUATION $\boxed{x^2 - 6x + 5 = 0}$

$x \quad -5 \quad -5x$

$x \quad -1 \quad -1x$

$-6x$ ✓

BRACKETS $\boxed{(x-5)(x-1)} = 0$

$x - 5 = 0$, $+5 \quad +5$, $x = 5$

$x - 1 = 0$, $+1 \quad +1$, $x = 1$

ROOTS $\boxed{x = 5}$ $\boxed{x = 1}$

NOW WE NEED TO GO THE OTHER WAY.

SO,

ROOTS $\rightarrow$ BRACKETS

BRACKETS. $\rightarrow$ EQUATION

EG. FORM THE QUADRATIC EQUATION WITH THE ROOTS 3 AND -1

THIS MEANS $x = 3$ $x = -1$

ROOTS $\rightarrow$

$x = 3$ | $x = -1$
$x - 3 = \cancel{3} - 3$ | $x + 1 = \cancel{-1} + 1$
$x - 3 = 0$ | $x + 1 = 0$

$\boxed{\text{WE NEED } = 0}$

THESE ARE NOW THE BRACKETS

BRACKETS $(x - 3)(x + 1) = 0$

FOIL

$x^2 + \underline{1x - 3x} - 3 = 0$

EQUATION $\rightarrow$ $\boxed{x^2 - 2x - 3 = 0}$

NIGHTMARE

SIMULTANEOUS EQUATIONS WITH 1 QUADRATIC.

eg SOLVE:

$$x^2 + y^2 = 10 \rightarrow \text{QUADRATIC}$$

$$x - y = 4 \rightarrow \text{LINEAR}$$

- THESE ARE LONG, BUT CAN COME UP IN DIFFERENT PARTS OF THE COURSE, SO WE HAVE TO BE ABLE TO DO THEM...

METHOD

1. WRITE EQUATIONS
2. ISOLATE ONE OF THE LETTERS FROM THE LINEAR.
3. SUBSTITUTE THIS INTO THE QUADRATIC.
4. THIS GIVES YOU A QUADRATIC EQUATION WITH 1 LETTER
5. SOLVE [2 ANSWERS]
6. DON'T FORGET TO GET THE OTHER LETTER IN EACH CASE...

- WE ARE BEING ASKED TO FIND THE POINTS OF INTERSECTION OF THE TWO FUNCTIONS / EQUATIONS.

- YOU WILL NEED TO DO A FEW BITS OF ROUGH WORK AT TIMES.

- IN THE MIDDLE, YOU WILL GET A QUADRATIC EQUATION. YOU CAN SOLVE THIS USING EITHER FACTORISING OR THE $-b$ FORMULA.

- SEE THE EXAMPLE ON THE NEXT PAGE.

① $x^2 + y^2 = 10$

② $x - y = 4$

② $x - \cancel{y} = 4$
$+\cancel{y} \quad +y$

ISOLATE ONE OF THE LETTERS

$\boxed{x = 4 + y}$ ③

SUB THIS INTO QUADRATIC.

① $x^2 + y^2 = 10$

$(4+y)^2 + y^2 = 10$

$16 + 8y + y^2 + y^2 = 10$
$-10 \qquad -10$

TIDY UP TO GET =0

$2y^2 + 8y + 6 = 0$

THIS IS YOUR NEW QUADRATIC EQUATION

RW

$(4+y)^2$

$(4+y)(4+y)$

$4(4+y) + y(4+y)$

$16 + 4y + 4y + y^2$

$16 + 8y + y^2$

METHOD 1 FACTORS

$2y^2 + 8y + 6 = 0$

$\div 2 \quad y^2 + 4y + 3 = 0$

$y \quad 3$

$y \quad 1$

$(y+3)(y+1) = 0$

$\boxed{y = -3}$ OR $\boxed{y = -1}$

$x = 4 + y$ ③ | $x = 4 + y$ ③

$x = 4 - 3 = 1$ | $x = 4 - 1 = 3$

$\boxed{x = 1}$ | $\boxed{x = 3}$

$(1, -3)$ | $(3, -1)$

METHOD 2 $-b$ FORMULA

$a = 2$

$b = 8$

$c = 6$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-8 \pm \sqrt{(8)^2 - 4(2)(6)}}{2(2)}$$

$$\frac{-8 \pm \sqrt{64 - 48}}{4}$$

$$\frac{-8 + \sqrt{16}}{4} \quad \text{OR} \quad \frac{-8 - 4}{4}$$

$\boxed{y = -1}$ OR $\boxed{y = -3}$

$x = 4 - 1$ | $x = 4 - 3$

$\boxed{x = 3}$ OR $\boxed{x = 1}$

$(3, -1)$ | (,)

IT CAN HELP TO DRAW A GRAPH

OR

YOU MIGHT BE ASKED TO USE A GRAPH TO SOLVE QUADRATIC EQUATIONS.

VERY IMPORTANT

"ROOTS" = ANSWERS = WHERE GRAPH CROSSES x-AXIS

WHERE 2 GRAPHS CROSS / TOUCH ARE CALLED "POINTS OF INTERSECTION"

METHOD

- EITHER DRAW THE GRAPH OR USE THE GRAPH GIVEN TO YOU.
- IDENTIFY THE "ROOTS" / "SOLUTIONS" / "ANSWERS" BY FINDING WHERE THE GRAPH CROSSES THE x-AXIS.
- THIS IS AN ESTIMATE - YOU MAY BE ASKED TO SUBSTITUTE YOUR ANSWER BACK IN TO THE ORIGINAL FUNCTION: REPLACE THE x IN THE FUNCTION WITH YOUR ANSWER(S) FOR x.

COMPLEX NUMBERS

eg $3 + 4i$ OR $a + bi$

3 ← REAL, $4i$ ← IMAGINARY; a ← REAL, bi ← IM

JUST LIKE ALGEBRA - EXCEPT $\boxed{i^2 = -1}$ OR $\boxed{i = \sqrt{-1}}$

INTRODUCTION

- WE NEED COMPLEX NUMBERS TO DEAL WITH SQUARE ROOTS OF NEGATIVE NUMBERS.
- TRY ENTERING $\sqrt{-4}$ INTO YOUR CALCULATOR → YOU WILL GET AN ERROR.
- WE HAVE $\boxed{i = \sqrt{-1}}$. SO

$$\boxed{\sqrt{-4}} = \underbrace{\sqrt{4}}_{\sqrt{4}=2} \times \sqrt{-1} = \boxed{2i}$$

- IN PRACTICE, A LOT OF WHAT WE DO WITH COMPLEX NUMBERS IS JUST NORMAL ALGEBRA EXCEPT:

EVERYWHERE YOU COME ACROSS i^2 REPLACE IT WITH -1

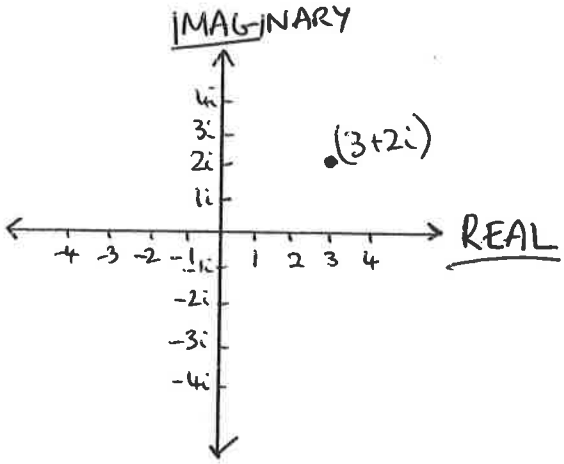
SO $3i^2 = -3$
$-5i^2 = +5$

HINT: IF YOU SEE i^2, CHANGE THE SIGN AND DROP THE i^2

ARGAND DIAGRAM

THIS IS WHERE WE PLOT COMPLEX NUMBERS AS CO-ORDINATES. (LIKE X AND Y AXES)

IT OFTEN HELPS US TO DRAW A PICTURE.



MODULUS

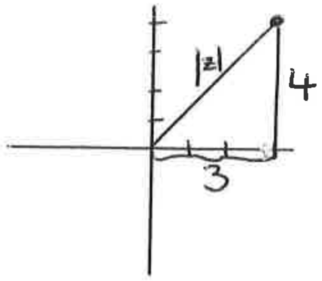
$|z|$ (THIS IS THE DISTANCE FROM THE "ORIGIN" TO THE "POINT" REPRESENTED ON THE ARGAND DIAGRAM).

BE VERY FAMILIAR WITH THE NOTATION / HOW THE MODULUS IS WRITTEN ...

1. PLOT THE POINT ON THE ARGAND DIAGRAM. (THIS CAN BE A VERY QUICK SKETCH)
2. DRAW A RIGHT-ANGLED TRIANGLE + LABEL SIDE LENGTHS
3. USE PYTHAGORAS

eg $z = 3 + 4i$

CALCULATE $|z|$ →



$$|z|^2 = 3^2 + 4^2$$
$$|z|^2 = 9 + 16$$
$$|z|^2 = 25$$
$$|z| = \sqrt{25} = \boxed{5}$$

. WE OFTEN USE THE LETTERS Z OR w TO REPRESENT COMPLEX NUMBERS.

eg $Z = 3 + 2i$

WHAT IS Z^2

ANS:

$$(3+2i)^2$$
$$= (3+2i)(3+2i)$$
$$= 9 + 6i + 6i + 4i^2$$
$$= 9 + 12i - 4$$
$$= \boxed{5 + 12i}$$

CONJUGATE

eg IF $Z = 3 + 5i$

$\downarrow$

$\bar{Z}$

YOU HAVE TO KNOW THE NOTATION

$\bar{Z} = 3 - 5i$

CHANGE THE SIGN OF THE IMAGINARY PART ONLY

eg ① $Z = 3 + 4i$

$\bar{Z} = 3 - 4i$

② $w = 2 - 5i$

$\bar{w} = 2 + 5i$

* THIS WILL BECOME VERY USEFUL FOR DIVIDING COMPLEX NUMBERS

INVESTIGATION : WHAT HAPPENS WHEN YOU MULTIPLY BY i ?

$i = \boxed{i}$

$i^2 = i \times i = i^2 = \boxed{-1}$

$i^3 = i \times i \times i$

$= (i \times i) \times i$

$= -1 \times i = \boxed{-i}$

$i^4 = i^3 \times i$

$= (-i) \times i = -i^2 = \boxed{1}$

$i^5 = i^4 \times i$

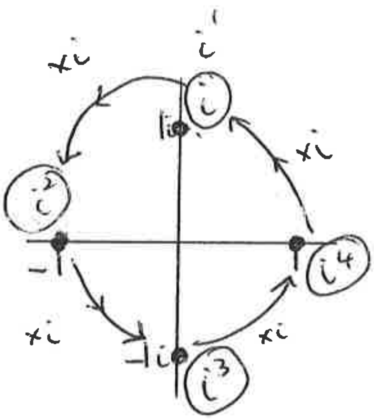
$= 1 \times i = \boxed{i}$

etc.

$$\begin{bmatrix} i = i \\ i^2 = -1 \\ i^3 = -i \\ i^4 = 1 \end{bmatrix}$$

REPEATS EVERY FOUR

$$\begin{bmatrix} i^5 = i \\ i^6 = -1 \\ i^7 = -i \\ i^8 = 1 \end{bmatrix}$$



SO, MULTIPLYING BY i IS A ROTATION BY $90°$

YOU SHOULD KNOW/ REMEMBER THIS ...

ADDING / SUBTRACTING COMPLEX NUMBERS

JUST LIKE IN ALGEBRA, WHEN WE CAN ONLY ADD "LIKE TERMS", WITH COMPLEX NUMBERS WE CAN ADD/SUBTRACT "REALS" AND "IMAGINARIES" SEPARATELY:

eg $3 + 4i + 2 - 3i$

REAL

$3+2$

IMAGINARY

$4i - 3i$

$= \boxed{5 + 1i}$

MULTIPLYING COMPLEX NUMBERS

BEHAVES EXACTLY LIKE ALGEBRA, EXCEPT $i^2 = -1$

EG.

① $3(4 - 5i)$

$= 12 - 15i$

② $2(3 + 2i) - 4(2 + i)$

$= 6 + 4i - 8 - 4i$

$= \boxed{-2}$

③ $3i(1 + 2i)$

$= 3i + 6i^2$

REMEMBER, $\boxed{i^2 = -1}$

$= 3i - 6$

$= -6 + 3i$

WE WRITE THE REAL BIT FIRST

④ $(-3 + 2i)(4 - 5i)$

FOIL

$= -12 + \underbrace{15i + 8i} - 10i^2$

$= -12 + 23i + 10$

$= -2 + 23i$

DIVIDING COMPLEX NUMBERS

eg $\dfrac{3+2i}{4-5i}$

① EASY !

IF IT'S JUST A REAL NUMBER ON THE BOTTOM. → DIVIDE EACH TOP TERM BY THE NUMBER ON THE BOTTOM

eg. ① $\dfrac{10+15i}{5} = \boxed{2+3i}$

② $\dfrac{23-17i}{4} = \boxed{\dfrac{23}{4} - \dfrac{17}{4}i}$

---

② HARD

IF THE BOTTOM HAS AN IMAGINARY PART → MAKE IT INTO THE "EASY" TYPE

HOW? BY MULTIPLYING TOP AND BOTTOM BY THE CONJUGATE OF THE BOTTOM

eg

COMPLEX NO.	CONJUGATE
$4+3i$	$4-3i$
$-2-4i$	$-2+4i$
$5-7i$	$5+7i$

ie CHANGE THE SIGN OF THE IMAGINARY PART

WHY? BECAUSE IT MAKES THE BOTTOM A REAL NUMBER

eg $\dfrac{5+5i}{1+2i}$

CONJUGATE OF $1+2i$ → $(1-2i)$

STEP 1 MULTIPLY TOP AND BOTTOM BY $(1-2i)$

$$\frac{(5+5i)(1-2i)}{(1+2i)(1-2i)}$$

STEP 2 LAY IT OUT LIKE THIS

TOPS

$(5+5i)(1-2i)$

$5(1-2i) + 5i(1-2i)$

$5 - 10i + 5i - 10i^2$

$5 - 5i + 10$

TOP : $\boxed{15 - 5i}$

BOTS

$(1+2i)(1-2i)$

$1(1-2i) + 2i(1-2i)$

$1 - 2i + 2i - 4i^2$

$1 + 4$

BOT : $\boxed{5}$

STEPS 3 + 4 : PUT $\dfrac{\text{TOP}}{\text{BOT}}$ AND DIVIDE

$$\frac{15-5i}{5} = \boxed{3 - i}$$

USING COMPLEX NUMBERS FOR QUADRATIC EQUATIONS

eg $z^2 - 6z + 34 = 0$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$

$b = -6$

$c = 34$

THIS CAN'T BE SOLVED USING <u>REAL</u> NUMBERS ALONE!

SUB IN

$$\frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(34)}}{2(1)}$$

<u>RW</u> DO THE BIT UNDER THE SQUARE ROOT SIGN SEPARATELY.

$\sqrt{(-6)^2 - 4(1)(34)}$

$= \sqrt{36 - 136}$

$= \sqrt{-100}$

$= \sqrt{100} \times \sqrt{-1}$

$= \boxed{10\,i}$

$$\frac{6 \pm 10i}{2}$$

$\boxed{3 + 5i}$ OR $\boxed{3 - 5i}$

THIS WILL ALWAYS HAPPEN. IF THE 2 ANSWERS ARE COMPLEX NUMBERS, ONE WILL BE THE "CONJUGATE" OF THE OTHER

YOU WILL NEED TO PRACTISE LOTS OF THESE

eg ① $z^2 - 10z + 29 = 0$

② $z^2 + 2z + 10 = 0$

③ $z^2 - 12z + 37 = 0$

④ $z^2 - 2z + 17 = 0$

Course 1 Algebra Functions Differentiation

Complex Numbers

Question 1

Joseph is doing a training session. During the session, his heart-rate, $h(x)$, is measured in beats per minute (BPM). For part of the session, $h(x)$ can be modelled using the following function:

$$h(x) = -0{\cdot}38x^3 + 2{\cdot}6x^2 - 0{\cdot}13x + 158$$

where x is the time, in minutes, from the start of the session, and $0 \leq x \leq 6$, $x \in \mathbb{R}$.

(e) Joseph has a smart watch that beeps every 15 seconds during the session.
It beeps for the first time at exactly $2{:}\,55$ p.m., as Joseph starts his session.
It beeps for the last time at exactly $3{:}\,23$ p.m., as Joseph finishes his session.

Work out how many times, in total, the smart watch beeps during the session, including the first and last beep.

(f) Solve the equation

$$h'(x) = -\,1{\cdot}14x^2 + 5{\cdot}2x - 0{\cdot}13 \;\; = 0$$

to find how long after the start of the session Joseph's heart-rate is at a maximum, for $0 \leq x \leq 6$, $x \in \mathbb{R}$. Give your answer in minutes, correct to 2 decimal places.

Question 2

Question 2 **(25 marks)**

(a) Solve the equation:

$$\frac{9x-6}{2}=\frac{3x-14}{3}+\frac{9x}{4}.$$

(b) Solve the simultaneous equations:

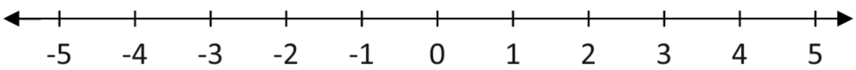
$$3x - y = 4$$
$$4x^2 - 3xy = 4.$$

Question 3

Question 6 **(25 marks)**

(a) Solve the following inequality for $x \in \mathbb{R}$ **and** show your solution on the numberline below:

$$2(3 - x) < 8.$$



(b) Solve for x:

$$2^{2x-1} = 64.$$

Question 4

Question 3 **(25 marks)**

(a) Solve the equation $2x^2 - 7x - 3 = 0$. Give each answer correct to 2 decimal places.

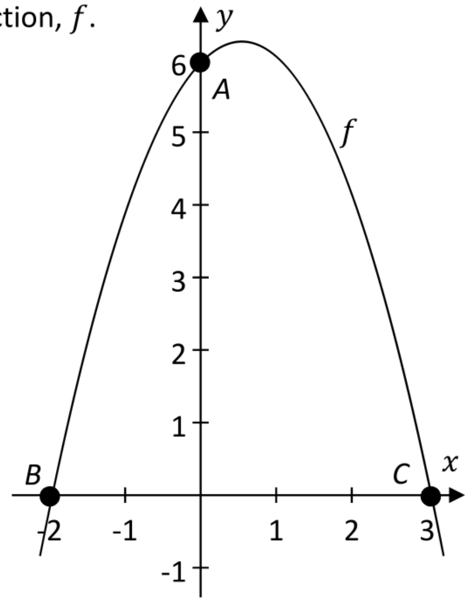
(b) Solve the simultaneous equations below to find the value of a and the value of b.

$$\begin{aligned} 2a + 3b &= 15 \\ 5a + b &= -8 \end{aligned}$$

Question 5

Question 5 **(25 marks)**

The diagram on the right shows the graph of a quadratic function, f.



(a) Write down the co-ordinates of A, B, and C.

$A = (\quad , \quad)$

$B = (\quad , \quad)$

$C = (\quad , \quad)$

(b) Show that the function can be written as $f(x) = -x^2 + x + 6$.

(c) Show, **using calculus**, that the maximum point of $f(x)$ is (0·5, 6·25).

Question 6

The function $f: x \mapsto x^3 + x^2 - 2x + 7$ is defined for $x \in \mathbb{R}$.

(a) Find the coordinates of the point at which the graph of f cuts the y-axis.

(b) Verify, using algebra, that the point $A(1, 7)$ is on the graph of f.

(c) **(i)** Find $f'(x)$, the derivative of $f(x)$.
Hence find the slope of the tangent to the graph of f when $x = 1$.

$f'(x) =$

Slope =

(ii) Hence, find the equation of the tangent to the graph of f at the point $A(1, 7)$.

Question 10 **(50 marks)**

Keith plays hurling.

(a) During a match, Keith hits the ball with his hurl.
The height of the ball could be modelled by the following quadratic function:

$$h = -2t^2 + 5\mathrm{t} + 1{\cdot}2$$

where h is the height of the ball, in metres, t seconds after being hit, and $t \in \mathbb{R}$.

(i) How high, in metres, was the ball when it was hit (when $t = 0$)?

(ii) The ball was caught after $2{\cdot}4$ seconds.
How high, in metres, was the ball when it was caught?

(iii) When the ball passed over the halfway line, it was at a height of 3·2 metres and its height was decreasing.

How many seconds after it was hit did the ball pass over the halfway line? Remember that $h = -2t^2 + 5\text{t} + 1{\cdot}2$.

Answer: ____________

(iv) Find $\frac{dh}{dt}$ **and** hence find how long it took the ball to reach its greatest height.

Give your answer in seconds.

$\frac{dh}{dt} =$ ________________________________

Length of time = _______________

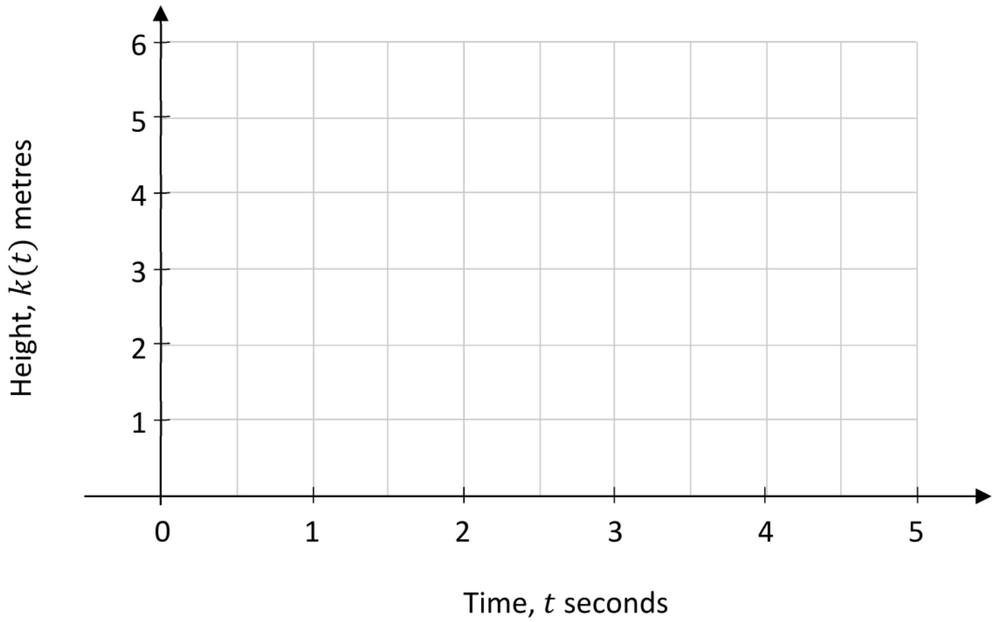
This question continues on the next page.

(b) Later in the game, Keith hit the ball again. This time, the height of the ball t seconds after it was hit could be modelled by a different quadratic function, $y = k(t)$, where k is in metres.

This time, the ball was 1 metre high when Keith hit it.
Its greatest height was 5 metres, which it reached after 2 seconds.
It hit the ground without being caught.

Using the information above, write down the co-ordinates of three points that **must** be on the graph of $y = k(t)$, **and** draw the graph of $y = k(t)$ on the axes below, from when the ball is hit until it hits the ground.

Points: $(\quad , \quad)$, $(\quad , \quad)$, and $(\quad , \quad)$



Question 6 **(25 marks)**

(a) **(i)** Differentiate the function $f(x) = 4x^3 - 3x^2 + x - 7$, where $x \in \mathbb{R}$, with respect to x.

(ii) Find the slope of the tangent to the graph of $f(x) = 4x^3 - 3x^2 + x - 7$ at the point $(1, -5)$.

(iii) Hence find the equation of the tangent to the graph at this point.

(b) The function $g(x) = 2x^2 + px + q$, where $p, q \in \mathbb{Z}$, and $x \in \mathbb{R}$.
Given that $g(2) = 6$ and $g'(3) = 9$, find the value of p and the value of q.

Note: $g'(3)$ is the value of the derivative of $g(x)$ at $x = 3$.

$p =$ $q =$

Question 9

Question 8 **(65 marks)**

The amount, in appropriate units, of a certain medicinal drug in the bloodstream t hours after it has been taken can be estimated by the function:

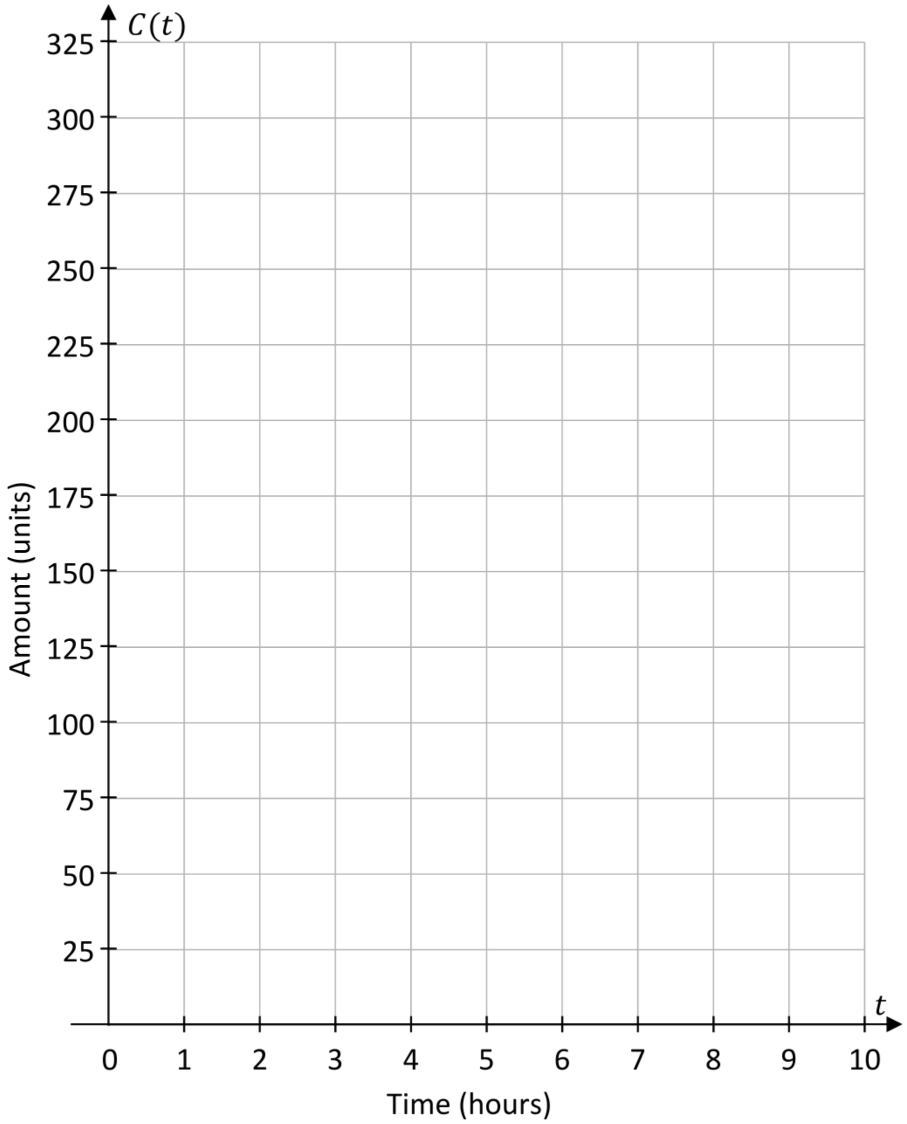
$$C(t) = -t^3 + 4{\cdot}5t^2 + 54t, \quad \text{where} \quad 0 \le t \le 9, \ t \in \mathbb{R}.$$

(a) Use the drug amount function, $C(t)$, to show that the amount of the drug in the bloodstream 4 hours after the drug has been taken is 224 units.

(b) Use the function $C(t)$ to complete the table below.

t (Hours)	0	1	2	3	4	5	6	7	8	9
$C(t)$ (Units)	0	57·5			224					

(c) Draw the graph of the function $C(t)$ for $0 \leq t \leq 9$ where $t \in \mathbb{R}$.



(d) Use your graph to estimate each of the following values.
In each case show your work on the graph above.

(i) The amount of the drug in the bloodstream after $3\frac{1}{2}$ hours.

(ii) How long after taking the drug will the amount of the drug in the bloodstream be 100 units?

(e) **(i)** Use the drug amount function $C(t) = -t^3 + 4{\cdot}5t^2 + 54t$ to find, in terms of t, the rate at which the drug amount is changing after t hours.

(ii) Use your answer to part **e(i)** to find the rate at which the drug amount is changing after 4 hours.

(iii) Use your answer to part **e(i)** to find the maximum amount of the drug in the bloodstream over the first 9 hours.

(iv) Use your answer to part **e(i)** to show that the drug amount in the bloodstream is decreasing 7 hours after the drug has been taken. Explain your reasoning.

Show:

Explanation:

Question 10

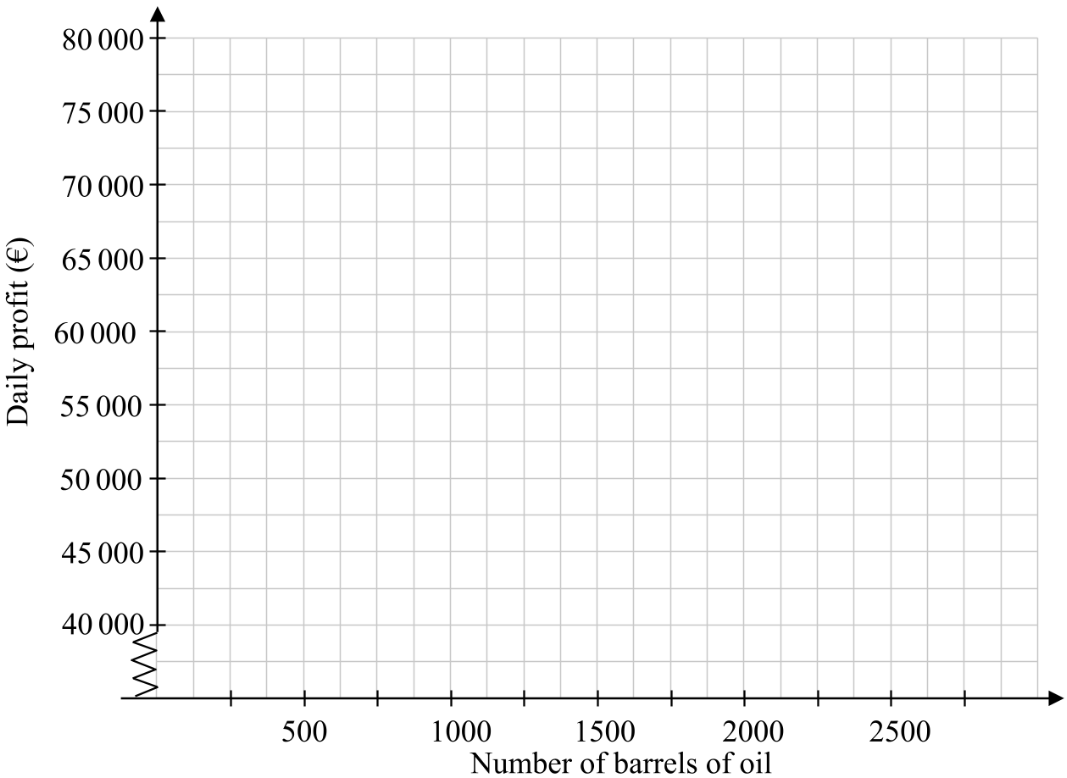
Question 8 **(50 marks)**

The daily profit of an oil trader is given by the profit function $p = 96x - 0{\cdot}03x^2$, where p is the daily profit, in euro, and x is the number of barrels of oil traded in a day.

(a) Complete the table below.

Number of barrels traded in a day	x	500	1000	1500	2000	2500
Daily profit (€)	p	40 500				

(b) Draw the graph of the trader's profit function on the axes below for $500 \leq x \leq 2500,\ x \in \mathbb{R}$.



(c) Use your graph to estimate:

(i) The daily profit when 1750 barrels are traded. Answer: ____________

(ii) The numbers of barrels traded when the daily profit is €60 000.

Answer: __________________ or _________________

(d) **(i)** Use calculus to find the number of barrels of oil traded that will earn the maximum daily profit.

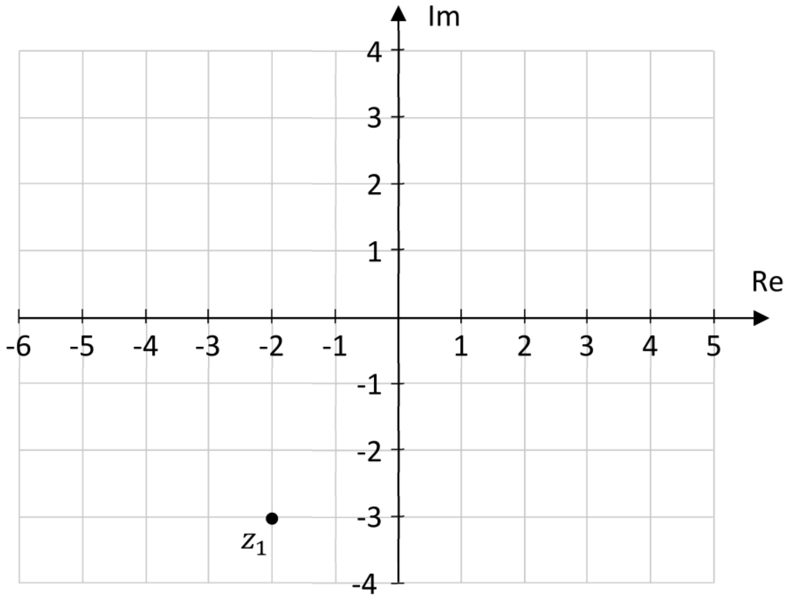
(ii) Find this maximum profit.

(e) The trader will not make a profit if he trades more than k barrels of oil in a day. Calculate the value of k.

Question 11

Question 1 **(30 marks)**

The complex number z_1 is shown on the Argand diagram below.



(a) Using the Argand diagram:

(i) write down the values of z_1 and $\overline{z_1}$, where $\overline{z_1}$ is the complex conjugate of z_1

$z_1 =$ ☐ $\overline{z_1} =$ ☐

(ii) plot and label $\overline{z_1}$ on the Argand diagram above.

z_2 and z_3 are two other complex numbers.

$z_2 = -5 + 3i$ and $z_3 = 4 - 2i$, where $i^2 = -1$.

(b) Plot and label z_2 and z_3 on the Argand diagram on the previous page.

(c) Write $z_2 - z_3$ in the form $a + bi$, where $a, b \in \mathbb{R}$, $i^2 = -1$, **and** hence find $|z_2 - z_3|$.

$z_2 - z_3 =$ _______________

$|z_2 - z_3| =$ _______________

(d) Investigate if $z_3 = 4 - 2i$ is a solution of the equation $z^2 + 2iz - 7i = 0$.

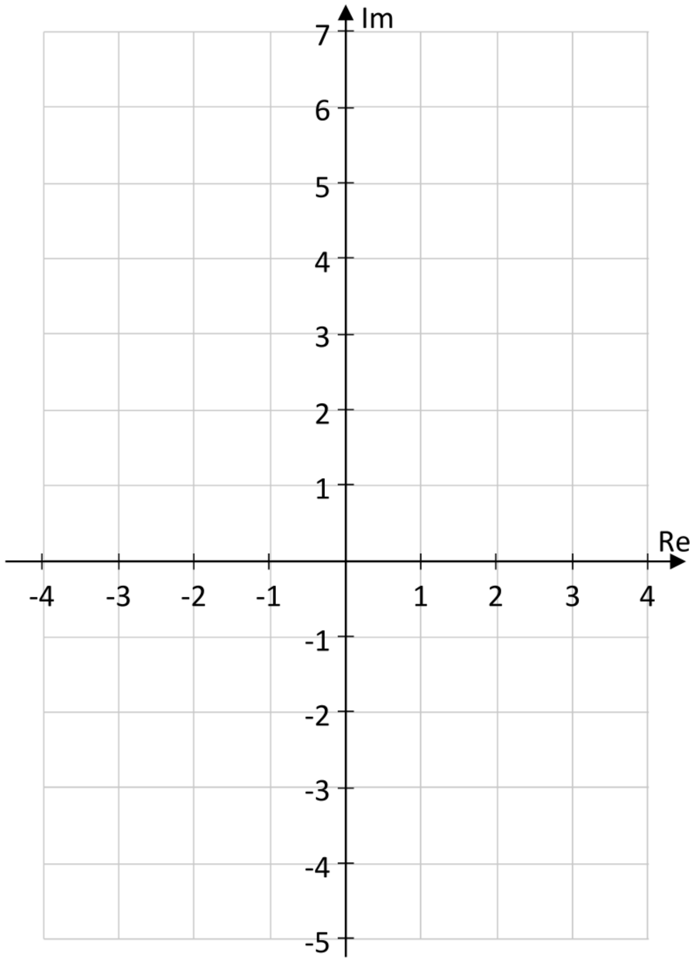
Conclusion: ___________________________________

Question 2 **(30 marks)**

$z_1 = -3 + 4i$ and $z_2 = 4 + 3i$,
where $i^2 = -1$.

(a) Plot and label z_1, z_2, and $z_1 + z_2$ on the Argand Diagram.

$z_1 + z_2 =$



(b) $z_3 = \frac{z_1}{z_2}$. Find z_3 in the form $a + bi$, where $a, b \in \mathbb{Z}$.

(c) Find $|\bar{z}_1 - z_2|$, where $\bar{z}_1$ is the complex conjugate of z_1.
Give your answer in the form $p\sqrt{q}$, where p and $q \in \mathbb{N}$.

Question 13

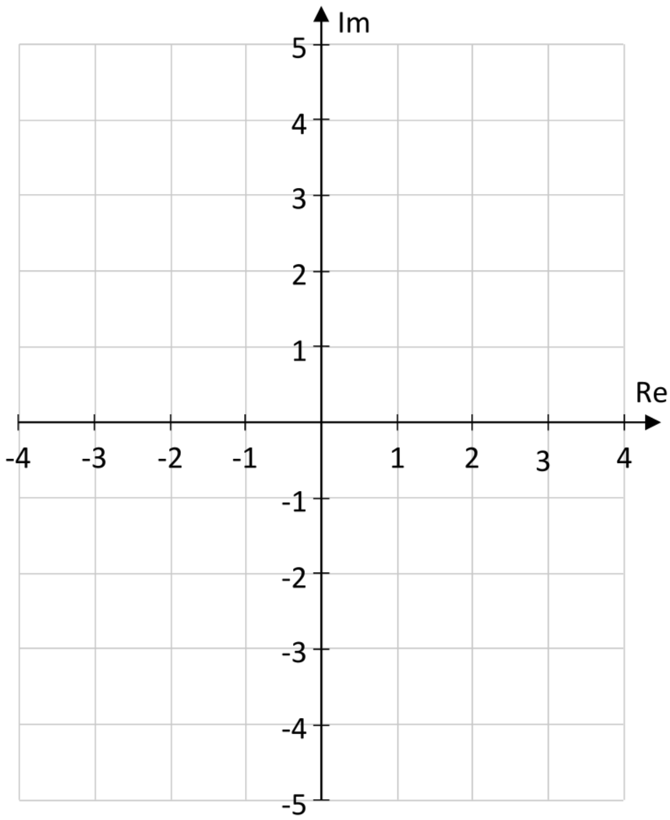
Question 3 **(25 marks)**

$z_1 = 3 - 4i,\ z_2 = -2 + i$ and $z_3 = 2iz_2$, where $i^2 = -1$.

(a) **(i)** Write z_3 in the form $a + bi$, where $a, b \in \mathbb{Z}$.

$z_3 = 2iz_2 =$

(ii) Plot z_1, z_2 and z_3 on the given Argand Diagram.
Label each point clearly.



(iii) Find $|z_1|$.

(b) If $z_1 \times z_4 = 29 + 3i$, write z_4 in the form $a + bi$, where $a, b \in \mathbb{R}$.

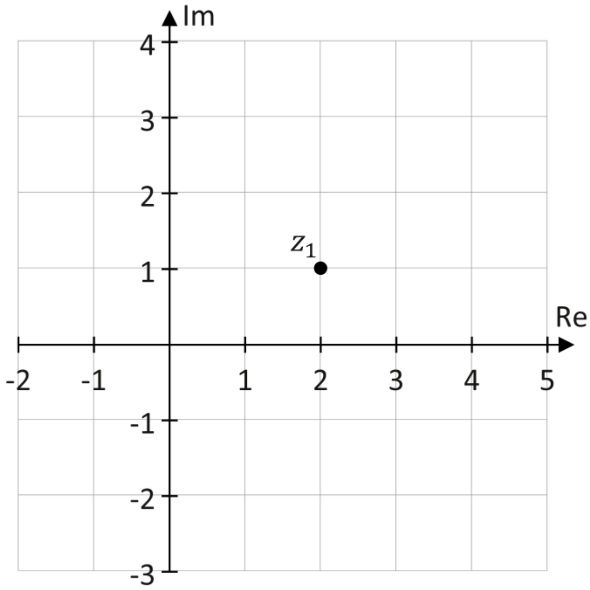
Question 14

Question 2 **(25 marks)**

The complex number $z_1 = 2 + i$, where $i^2 = -1$, is shown on the Argand Diagram below.

(a) **(i)** $z_2 = 2z_1$.

Find the value of z_2, and **plot and label** it on the Argand Diagram.



(ii) $\bar{z}_1$ is the complex conjugate of z_1.
Write down the value of $\bar{z}_1$, and **plot and label** it on the Argand Diagram.

$\bar{z}_1 =$

(iii) Investigate if $|z_2| = |z_1 + \bar{z}_1|$.

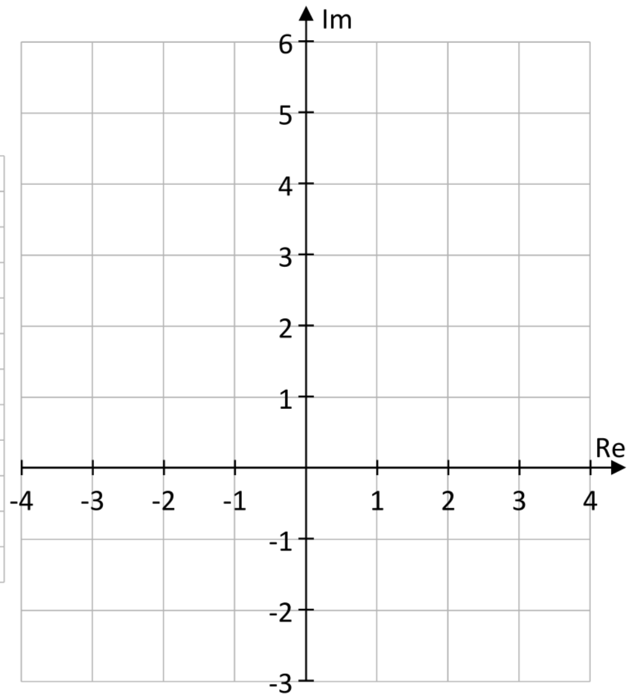
(b) Show that $z_1 = 2 + i$ is a solution of the equation $z^2 - 4z + 5 = 0$.

Question 15

Question 2 **(25 marks)**

$z_1 = -2 + 3i$ and $z_2 = -3 - 2i$, where $i^2 = -1$.
$z_3 = z_1 - z_2$.

(a) Plot z_1, z_2, and z_3 on the Argand Diagram.
Label each point clearly.



(b) Investigate if $|z_3| = |z_1| + |z_2|$.

Conclusion:

(c) $z_4 = \dfrac{z_1}{z_2}$. Write z_4 in the form $x + yi$, where $x, y \in \mathbb{R}$.

DIFFERENTIATION / CALCULUS

THE RATE OF CHANGE OF ONE VARIABLE COMPARED TO ANOTHER.

"THE MATHEMATICS OF CHANGE"

eg SPEED = HOW FAST YOUR DISTANCE IS CHANGING OVER TIME.

KEY WORDS:

- DERIVATIVE
- SLOPE OF TANGENT
- FIRST DERIVATIVE
- MAXIMUM / MINIMUM VALUES
- LIMIT.
- RATE OF CHANGE.

DIFFERENTIATION IS ALL ABOUT **SLOPES**

SLOPE = HOW FAST IS y CHANGING COMPARED TO HOW FAST IS x CHANGING $\dfrac{\text{(RISE)}}{\text{(RUN)}}$

THIS IS WHAT DIFFO IS ALL ABOUT.

ANY QUESTION THAT MENTIONS:

- SLOPES
- MAXIMUM / MINIMUM VALUES
  METHOD $\left[\frac{dy}{dx} = 0\right]$
- RATE OF CHANGE

YOU NEED TO IMMEDIATELY THINK **DIFFO**

BASIC RULE OF DIFFO

IN WORDS : MULTIPLY THE NUMBER BY THE POWER AND REDUCE THE POWER BY 1

eg ① $y = 3x^5$

$\frac{dy}{dx} = 15x^4$

② $y = 4x^2$

$\frac{dy}{dx} = 8x$

③ $y = 5x$

$\frac{dy}{dx} = 5$

* NOTE WHEN YOU DIFF A CONSTANT, IT BECOMES ZERO

eg $y = 7$

$\frac{dy}{dx} = 0$

NOTE THIS CAN BE DONE FOR MORE THAN ONE TERM

eg $y = x^5 + 4x^3 - 7x + 3$

$\downarrow$ DIFF $\quad\downarrow$ DIFF $\quad\downarrow$ DIFF $\quad\downarrow$ DIFF

$$\boxed{\frac{dy}{dx} = 5x^4 + 12x^2 - 7}$$

2ND DERIVATIVE

SOMETIMES YOU NEED TO DIFFO TWICE [ONLY IF YOU'RE ASKED TO...]

$y \xrightarrow{\text{DIFF}} \frac{dy}{dx} \xrightarrow{\text{DIFF}} \frac{d^2y}{dx^2}$

OR

$f(x) \rightarrow f'(x) \rightarrow f''(x)$

OR

DISTANCE $\xrightarrow{\text{DIFFO}}$ SPEED $\rightarrow$ ACCELERATION

EXTRA BIT

THEY MIGHT ASK YOU TO FIND $\frac{dy}{dx}$ AT A PARTICULAR POINT OR FOR A CERTAIN VALUE OF x.

eg IF $\boxed{y = x^2 - 5x}$, FIND THE VALUE OF $\frac{dy}{dx}$ WHEN $x = 1$

(COULD BE ANYTHING)

FIRST... FIND $\frac{dy}{dx} = 2x - 5$

THEN SUBSTITUTE. $\frac{dy}{dx} = 2(1) - 5$
@ $x=1$
$= \boxed{-3}$ (USE YOUR CALCULATOR)

DON'T SUBSTITUTE INTO y. IT MUST BE $\frac{dy}{dx}$.

REMEMBER, 2 DIFFERENT NOTATIONS

$y = f(x)$

$\frac{dy}{dx} = f'(x)$

$\frac{d^2y}{dx^2} = f''(x)$

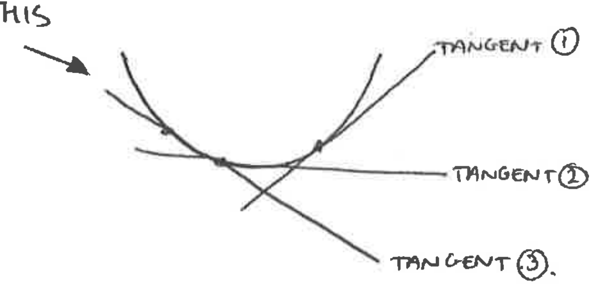
MAKE SURE YOU SUBSTITUTE INTO THE CORRECT ONE

ALL THIS TIME WHEN WE'VE BEEN JUST FOLLOWING SET ROUTINES OF HOW TO DIFFERENTIATE... WE HAVE BEEN WORKING OUT $\boxed{\text{SLOPES}}$

$\frac{dy}{dx}$ IS A ~~METHOD~~ FOR CALCULATING THE SLOPE OF THE TANGENT TO A CURVE AT ANY POINT OR FOR ANY VALUE OF x

eg. $y = x^2 - 2x + 3$

- IS A QUADRATIC FUNCTION.
- SO IT LOOKS LIKE THIS



- THE SLOPE OF THE TANGENT IS DIFFERENT/CHANGING FOR DIFFERENT VALUES OF x.

$$\boxed{\frac{dy}{dx} = 2x - 2}$$

(i) SLOPE at $x = 3$?

(ii) SLOPE at $(-1, 6)$?

(i) $\left.\frac{dy}{dx}\right|_{@\, x=3} = 2(3) - 2 = \boxed{4}$

(ii) $\left.\frac{dy}{dx}\right|_{@\, x=-1} = 2(-1) - 2 = \boxed{-4}$

IS IT INCREASING OR DECREASING ???...

IF $\frac{dy}{dx}$ IS:

POSITIVE $\longrightarrow$ INCREASING

NEGATIVE $\longrightarrow$ DECREASING

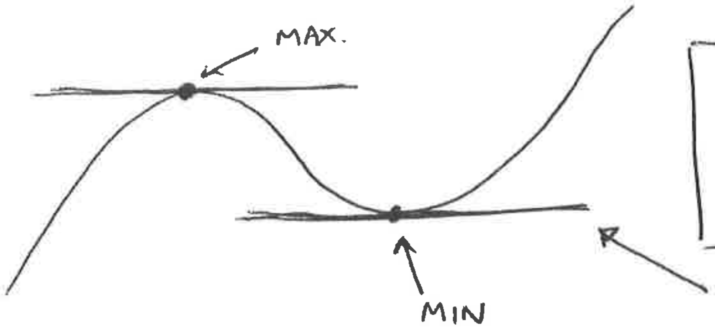
DIFFICULT QUESTIONS

- THEY MIGHT ASK YOU TO FIND A POINT WHERE THE SLOPE IS A CERTAIN VALUE
- TO DO THIS...

  FIND $\frac{dy}{dx}$ AND MAKE IT = TO THE SLOPE THEY GAVE YOU.

1 **TURNING POINTS** MAXIMUM / MINIMUM

eg



REMEMBER :

$$\boxed{\text{MAX/MIN} \quad \frac{dy}{dx} = 0}$$

SLOPE = 0 (HORIZONTAL LINE)

- AT THE "TURNING POINTS", THE FUNCTION CHANGES FROM INCREASING TO DECREASING, OR VICE VERSA
- AT THESE POINTS, THE SLOPE OF THE TANGENT = 0

i.e $\boxed{\frac{dy}{dx} = 0}$

QUADRATIC

(EASY) – 1 TURNING POINT

eg. $y = 2x^2 + 8x - 5$

$\frac{dy}{dx} = 4x + 8$

$\frac{dy}{dx} = 0$

SO $4x + 8 = 0$

$\quad -8 \quad -8$

$4x = -8$

$\boxed{x = -2}$

NOW WE NEED TO FIND y.

$y = 2x^2 + 8x - 5$

$x=-2$] $y = 2(-2)^2 + 8(-2) - 5 = -13$

SO $\boxed{(-2, -13)}$ IS THE MIN

CUBIC (HARD)
- 2 TURNING POINTS
- SOLVE QUADRATIC EQUATION.

eg $y = x^3 - 6x^2 + 9x - 10$

$\frac{dy}{dx} = 3x^2 - 12x + 9 = 0$

$\div 3 \qquad x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x = 1 \qquad x = 3$

$\boxed{y = x^3 - 6x^2 + 9x - 10}$

$x=1$: $y = (1)^3 - 6(1)^2 + 9(1) - 10$

$y = -6$

$(1, -6)$

MIN

$x=3$: $y = (3)^3 - 6(3)^2 + 9(3) - 10$

$y = 26$

$(3, 26)$

THIS IS MAX BECAUSE 26 IS HIGHER THAN −6.

RATES OF CHANGE / DISTANCE, SPEED, ACCELERATION

ANOTHER WAY TO THINK OF DIFFERENTIATION IS HOW FAST IS ONE VARIABLE (USUALLY y) CHANGING, COMPARED TO ANOTHER (USUALLY x)

SPEED = HOW FAST IS MY **DISTANCE** CHANGING OVER **TIME**?

ACCELERATION = HOW FAST IS MY **SPEED** CHANGING OVER **TIME**?

SO. IF I DIFF **DISTANCE** I GET **SPEED**

AND IF I DIFF **SPEED** I GET **ACCELERATION**

JUST REMEMBER THIS,

$$\boxed{\text{DISTANCE}} \xrightarrow{\text{DIFF}} \boxed{\text{SPEED}} \xrightarrow{\text{DIFF}} \boxed{\text{ACCELERATION}}$$

eg

DISTANCE $= t^3 - 2t^2 + 3t$ ← [THIS IS A FORMULA FOR DISTANCE]

↓ DIFF

SPEED $= 3t^2 - 4t$ ← [FORMULA FOR SPEED]

↓ DIFF

ACCELERATION $= 6t$ ← [FORMULA FOR ACCELERATION]

SO IF I WANT THE **SPEED** AFTER 3 SECONDS, I USE THE SPEED FORMULA.

$$\text{SPEED} = 3t^2 - 4t$$
$$[t=3] \quad = 3(3)^2 - 4(3)$$
$$= 27 - 12 = \boxed{15 \text{ m/s}}$$

IF A QUESTION IS USING **HEIGHT** FOR DISTANCE, THEY OFTEN ASK ABOUT **MAXIMUM** HEIGHT (REMEMBER MAX $\Rightarrow \frac{dy}{dx} = 0$?)

AT THE HIGHEST POINT, $\boxed{\text{SPEED} = 0}$ → SOLVE.

LIMITS

(LOOKS WEIRD – IT'S ACTUALLY VERY EASY!)

- WE SOMETIMES NEED TO KNOW THE "LIMIT" OF A FUNCTION FOR A PARTICULAR VALUE OF x.
- THIS SIMPLY MEANS: WHAT IS THE VALUE OF THE FUNCTION AT THIS POINT?
- THIS USUALLY MEANS SUBSTITUTE THE VALUE OF x INTO THE FUNCTION.

eg $$\lim_{x \to 3} (x+4)$$

SUBTITUTE 3 IN FOR x

$$= 3+4$$

$$= 7$$

THIS MEANS / WE SAY: "WHAT IS THE "LIMIT" OF $x+4$ WHERE $x=3$"

- IT CAN BE MORE DIFFICULT IF THE FUNCTION IS "UNDEFINED" AT THAT POINT. (ie. IF SUBSTITUTION DOESN'T WORK)

eg $$\lim_{x \to 4} \frac{x^2-16}{x-4}$$

IF SUBSTITUTION DOESN'T WORK ⇒ TIDY UP FIRST

- IF WE SUBSTITUTE $x=4$ INTO THE BOTTOM, WE WOULD BE DIVIDING BY $4-4$ WHICH IS 0. → WHICH IS NOT ALLOWED!!
- INSTEAD, FACTORISE + TIDY UP FIRST

ie x^2-16 IS $(x-4)(x+4)$ [D.O.T.S]

SO $\frac{\cancel{(x-4)}(x+4)}{\cancel{(x-4)}} = x+4$ AND $\lim_{x \to 4} x+4 = \boxed{8}$

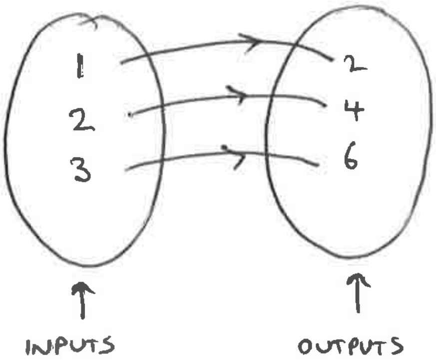
FUNCTIONS

[INPUTS → OUTPUTS]

A **FUNCTION** IS LIKE A "RULE", OR A "MACHINE". FOR **EVERY** **INPUT**, A FUNCTION GIVES US A PARTICULAR **OUTPUT**

WAYS OF SHOWING A FUNCTION:

MAPPING DIAGRAM



DOMAIN
ALL OF THE INPUTS

RANGE
ALL OF THE OUTPUTS

TABLE

INPUT	FUNCTION (RULE)	OUTPUT
-1	ADD 3	2
0	ADD 3	3
1	ADD 3	4
2	ADD 3	5

COUPLES / PAIRS

$$\{(1,2)\ (2,4)\ (3,6)\}$$

INPUT OUTPUT

(x, y)

FUNCTION NOTATION

$$f(x) = x^2$$

$$f(1) = (1)^2 = 1$$

$$f(2) = (2)^2 = 4$$

$$f(7) = (7)^2 = 49$$

INPUT OUTPUT

IMPORTANT NOTES / TERMS

THESE MEAN SAME THING.

x = INPUT

y = OUTPUT → ALSO $\boxed{y = f(x)}$

DOMAIN = ALL THE INPUTS

RANGE = ALL THE OUTPUTS

CODOMAIN = ALL THE "POSSIBLE" OUTPUTS

TWO INPUTS COULD HAVE THE SAME OUTPUT.

eg $f(x) = x^2$

DIFFERENT INPUTS

$f(1) = (1)^2 = 1$

AND

$f(-1) = (-1)^2 = 1$

SAME OUTPUTS

BUT

ONE INPUT CAN'T HAVE TWO DIFFERENT OUTPUTS.

i.e. THERE IS ONLY ONE OUTPUT FOR EACH INPUT.

SUBSTITUTION :

- IF YOU'RE GIVEN THE FUNCTION eg $f(x) = 2x + 3$
- REPLACE THE x WITH EACH INPUT (IN A BRACKET)

SO $f(2) = 2(2) + 3$

$f(2) = 7$

$(2, 7)$

COMPOSITE FUNTIONS

WHERE WE HAVE TO DO ONE FUNCTION AFTER ANOTHER...

eg $f(g(x))$ OR $f \circ g(x)$

eg $f(x) = 3x + 4$ AND $g(x) = x^2$

FIND (i) $f \circ g(2)$

(ii) $g(f(2))$

(i) $f \circ g(2)$ ← MEANS f AFTER $g(2)$

← SO DO $g(2)$ FIRST, THEN DO f TO YOUR ANSWER.

$g(2) = (2)^2$

$= 4$

$f(4) = 3(4) + 4$

$= \boxed{16}$

(ii) $g(f(2))$ ← MEANS g OF $f(2)$

← SO DO $f(2)$ FIRST, THEN DO g TO YOUR ANSWER

$f(2) = 3(2) + 4$

$f(2) = 10$

$g(f(2)) = g(10) = (10)^2$

$= \boxed{100}$

IN GENERAL **THE ORDER MATTERS**

$g \circ f(x)$ IS <u>NOT</u> THE SAME AS

$f \circ g(x)$

GRAPHING FUNCTIONS

- FUNCTIONS CAN BE GRAPHED ON THE X-AXIS AND Y-AXIS.

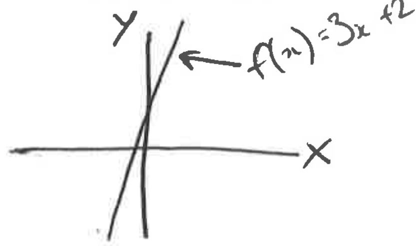
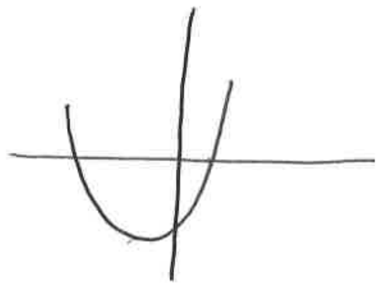
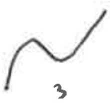
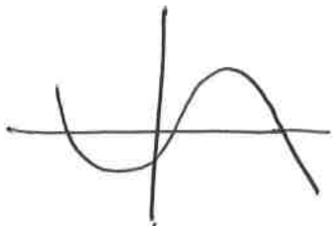
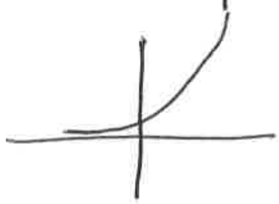
- REMEMBER THAT A FUNCTION IS A RULE THAT "maps" A PARTICULAR INPUT TO A PARTICULAR OUTPUT

  INPUT ↑ x OUTPUT ← y

- THESE CAN BE WRITTEN AS PAIRS OF INPUTS + OUTPUTS. (x, y) OR $(x, f(x))$

- THESE POINTS CAN THEN BE PLOTTED ON X-AXIS / Y-AXIS, MAKING A CERTAIN SHAPE

- WE NEED TO BE FAMILIAR WITH 4 PARTICULAR TYPES OF FUNCTION:

	eg	PICTURE
① **LINEAR** (JUST x - NO x^2's etc) LOOKS LIKE A LINE	$y = 3x + 2$ OR $f(x) = 3x + 2$	
② **QUADRATIC** (x^2 IN IT) LOOKS LIKE ∪ (x^2) OR ∩ ($-x^2$)	$f(x) = x^2 + 3x - 4$	
③ **CUBIC** (x^3 IN IT) LOOKS LIKE  ($-x^3$) OR  (x^3)	$f(x) = x^3 - 3x^2 + 2x + 4$	
④ **EXPONENTIAL** (x IN THE POWER)	$f(x) = 2(3^x)$	

TRANSFORMATIONS

[WHAT HAPPENS WHEN WE CHANGE THE ORIGINAL FUNCTION?]

[USE THE GRAPHING APPLETS ON THE WEBSITE]

- LINEAR ~ [EASY] – WE KNOW FROM CO-ORDINATE GEOMETRY THAT

$$y = \underset{\text{SLOPE}}{mx} + \underset{\text{Y-INTERCEPT}}{c}$$

SO IN OUR FUNCTIONS, IF WE CHANGE [eg $f(x) = 3x+2$] THE NUMBERS, WE ARE ALTERING THE SLOPE OR THE Y-INTERCEPT.

- REMEMBER, PARALLEL LINES HAVE THE SAME SLOPE

- IF WE CHANGE THE "m" PART, THE SLOPE CHANGES
- IF WE PLAY AROUND WITH THE "+c" PART THE GRAPH MOVES UP OR DOWN

- QUADRATICS $y = ax^2 + bx + c$

– IF WE CHANGE a, THE GRAPH GETS STEEPER/NARROWER

– IF WE CHANGE c, THE GRAPH MOVES UP/DOWN

(DON'T WORRY ABOUT b)

$$y = (x+b)^2$$

– CHANGING b MOVES THE GRAPH LEFT/RIGHT.

CUBICS

$$y = ax^3 + bx^2 + cx + d$$

- CHANGING d MOVES GRAPH ↑ OR ↓
- CHANGING a MAKES IT TALL + SKINNY OR SHORT + FAT.

(DON'T WORRY ABOUT b AND c)

$$y = (x + b)^3$$

→ AS WITH QUADRATICS, b MOVES THE GRAPH ← OR →

EXPONENTIAL

$$y = ak^x + b$$

→ CHANGING a MAKES IT STEEPER

→ CHANGING b MOVES IT ↑ OR ↓

→ CHANGING x [THE POWER] MOVES GRAPH ← OR →

FOR ALL THESE TRANSFORMATIONS, TRY MAKING A TABLE + PLOTTING NEW GRAPHS TO GET AN IDEA OF THE NEW SHAPE

INDICES

THIS IS ANOTHER WORD FOR "POWERS" eg 2^6

$2 \times 2 \times 2 \times 2 \times 2 \times 2$

- WHAT DO INDICES MEAN ?

BASE → 2^6 ← POWER

THE "BASE" MULTIPLIED BY ITSELF THE NUMBER OF TIMES INDICATED BY THE "POWER"

THIS MEANS → $\underbrace{2 \times 2 \times 2 \times 2 \times 2 \times 2}$

TWO MULTIPLIED BY ITSELF 6 TIMES.

- THIS IS NOT THE SAME AS THIS

2^6 ← DO NOT MAKE THIS MISTAKE → 2×6.

eg $3^4 = 3 \times 3 \times 3 \times 3$

$5^3 = 5 \times 5 \times 5$

$(-4)^2 = (-4) \times (-4)$

$a^5 = a \times a \times a \times a \times a$

MULTIPLYING POWERS / INDICES

eg $5^3 \times 5^4$?

THIS MEANS THIS MEANS

$(5 \times 5 \times 5) \times (5 \times 5 \times 5 \times 5)$

WHICH MEANS / IS THE SAME AS

$5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^7$

SO, $5^3 \times 5^4 = 5^7$

IF BASE IS THE SAME

MULTIPLYING

1 RULE

ADD THE "POWERS"

eg $a^2 \times a^3 = a^5$

DIVIDING POWERS / INDICES

eg $\dfrac{6^5}{6^3} = \dfrac{6 \times 6 \times 6 \times 6 \times 6}{6 \times 6 \times 6}$

NOW WE CAN "CANCEL" THE 6'S ON TOP AND BOTTOM.

$\dfrac{\cancel{6} \times \cancel{6} \times \cancel{6} \times 6 \times 6}{\cancel{6} \times \cancel{6} \times \cancel{6}} = 6^2$

So $\dfrac{6^5}{6^3} = 6^2$

2 DIVIDING / FRACTION RULE

SUBTRACT THE POWERS.

YOU NEED TO KNOW THESE RULES, AND EQUALLY IMPORTANTLY, YOU NEED TO KNOW **WHY** THEY WORK.

A POWER TO A POWER (BRACKETS)

eg $(2^3)^4$

MEANS $(2\times 2\times 2)^4$, SO $(2\times 2\times 2)\times(2\times 2\times 2)\times(2\times 2\times 2)\times(2\times 2\times 2)$

$$(2^3)^4 = 2^{12}$$

3 RULE
MULTIPLY THE POWERS

$$(\text{ANYTHING})^0 = 1$$

4 YOU NEED TO JUST REMEMBER THIS ONE. eg.

$5^0 = 1$; $3^0 = 1$; $2^0 = 1$

WHY ?

eg $\frac{3^7}{3^7}$ REMEMBER RULE 2 ? (SUBTRACT THE POWERS)

SO $\frac{3^7}{3^7} = 3^{7-7} = 3^0$

AND WE ALSO KNOW, $\frac{\text{ANYTHING}}{\text{ITSELF}} = 1$

eg $\frac{5}{5} = 1$; $\frac{2}{2} = 1$; $\frac{-4}{-4} = 1$

$\frac{3^7}{3^7} = 3^0$

$\frac{3^7}{3^7} = 1$

SO $3^0 = 1$

NEGATIVE POWERS

eg ⑤ $\boxed{3^{-4} = \frac{1}{3^4}}$

YOU NEED TO KNOW THIS BACKWARDS.

SO $\frac{1}{a^5} = a^{-5}$

FRACTIONAL POWERS = ROOTS eg $\boxed{16^{\frac{1}{2}} = \sqrt{16}}$ ⑥

$\boxed{a^{\frac{1}{k}} = \sqrt[k]{a}}$ " k^{TH} ROOT " [BUTTON ON CALCULATOR $\sqrt[\blacksquare]{\square}$]

$a^{1/2} = \sqrt{a}$

THERE ARE OTHER LAWS OF INDICES, BUT THESE ARE THE MAIN ONES ....

THE LAWS / RULES ARE ON p 21 OF THE TABLES BOOK. (BUT IT IS SLIGHTLY CONFUSING AND BADLY WRITTEN)

THE MOST IMPORTANT THING IS TO UNDERSTAND WHAT INDICES / POWERS **MEAN** AND BE ABLE TO WORK OUT QUESTIONS YOURSELF

eg $3^4 \times 3^3$ IF I FORGET THE RULE, THIS MEANS

$$\underbrace{3 \times 3 \times 3 \times 3}_{3^4} \times \underbrace{3 \times 3 \times 3}_{3^3}$$

WHICH IS $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 3^7$

EQUATIONS WITH INDICES

- THESE ARE TRICKY, BUT NOT IMPOSSIBLE.
- MAIN RULE:
  - GET SAME "BASE" ON BOTH SIDES
  - LET POWER = POWER

eg $$2^x = 2^5$$ ← SAME BASE

SO $x = 5$

- THEY ARE USUALLY NOT THIS EASY...

eg $$2^x = 32$$ ← NOT ~~PROBLEM~~ SAME BASE

REWRITE AS $32 = 2^5$

$$2^x = 2^5$$ ← SAME BASE!

NOW $x = 5$

- HARD EXAMPLE $$3^{3x-1} = \left(\frac{27}{\sqrt{3}}\right)^5$$

$27 = 3^3$ $\quad \sqrt{3} = 3^{1/2}$

UGH... WE NEED THIS AS $3^{\text{SOMETHING}}$

SO. $$\frac{27}{\sqrt{3}} = \frac{3^3}{3^{1/2}}$$

WHICH EQUALS $3^{2\frac{1}{2}}$ [SUBTRACT POWERS]

NOW,

$$3^{3x-1} = \left(3^{2\frac{1}{2}}\right)^5$$

[BRACKETS => MULTIPLY POWERS]

$5 \times 2\frac{1}{2} = \frac{25}{2}$

$$3^{3x-1} = 3^{\frac{25}{2}}$$

SO $$3x - 1 = \frac{25}{2} \overset{\times 2}{\Longrightarrow} 6x - 2 = 25$$

$+2 \qquad +2$

$$6x = 27$$

$$27 \div 6 = 4\frac{1}{2}$$

SURDS ($\sqrt{\ }$) – SQUARE ROOTS WHICH DON'T HAVE A NICE ANSWER.

→ THESE ARE <u>IRRATIONAL</u> NUMBERS
eg $\sqrt{2}$ or $\sqrt{3}$

✱ **MOST IMPORTANT THING**

WHEN YOU "SQUARE" A SURD IT CANCELS OUT. (BECAUSE THEY ARE OPPOSITES OF EACH OTHER)

---

eg

$$\sqrt{9} = 3$$
$$3^2 = 9$$

→ SO $(\sqrt{9})^2 = 9$

AND $(\sqrt{7})^2 = 7$

AND ALSO $\sqrt{6^2} = 6$

OTHER STUFF TO KNOW

- $\sqrt{3} \times \sqrt{2} = \sqrt{6}$ [YOU CAN MULTIPLY]
- $\frac{\sqrt{9}}{\sqrt{3}} = \sqrt{\frac{9}{3}} = \sqrt{3}$ [YOU CAN DIVIDE]

LET YOUR CALCULATOR DO MOST OF THESE QUESTIONS FOR YOU.

- IF YOU HAVE AN <u>EQUATION</u> WITH SURDS, **SQUARE BOTH SIDES** TO GET RID OF THE SURDS.

- IF A QUESTION SAYS "IN SURD FORM" IT MEANS IT HAS A $\sqrt{\ }$ SIGN IN IT.